Nurbs function:

$$R_i(x) = \frac{N_i(x)}{\sum_{j=1}^{N} w_j N_j(x)} w_i$$

Let

$$W = \sum_{j=1}^{N} w_j N_j(x)$$

Nurbs derivative:

$$R_i'(x) = \frac{N_i' W - N_i W'}{W^2} w_i = \frac{H}{W^2} w_i$$

Nurbs second derivative:

$$R_i''(x) = \frac{H' W - 2HW'}{W^3} = \frac{G}{W^3} w_i$$

where

$$H' = N_i'' W - NW''$$

Third derivative:

$$R_i'''(x) = \frac{G' W - 3GW'}{W^4} w_i$$

where

$$G' = H'' W - 2HW'' - H' W'$$

and

$$H'' = N_i''' W + N_i'' W' - N' W'' - NW'''.$$

Nurbs 2D function:

$$R_{ij}(x,y) = \frac{N_i(x)N_j(y)}{\sum_k \sum_l N_k(x)N_l(y)w_{kl}} w_{ij} = R_{ij}(x,y) = \frac{N_i(x)N_j(y)}{W} w_{ij}$$

First derivative:

$$\frac{\partial R_{ij}}{\partial x} = \frac{N_i'(x)N_j(y)W - N_i(x)N_j(y)\frac{\partial W}{\partial x}}{W^2} w_{ij} = \frac{H_1}{W^2} w_{ij}$$
$$\frac{\partial R_{ij}}{\partial y} = \frac{N_i(x)N_j'(y)W - N_i(x)N_j(y)\frac{\partial W}{\partial y}}{W^2} w_{ij} = \frac{H_2}{W^2} w_{ij}$$

Second derivative:

$$\frac{\partial^2 R_{ij}}{\partial x^2} = \frac{\frac{\partial H_1}{\partial x}W - 2H_1\frac{\partial W}{\partial x}}{W^3} w_{ij} = \frac{G_1}{W^3} w_{ij}$$
$$\frac{\partial^2 R_{ij}}{\partial y^2} = \frac{\frac{\partial H_2}{\partial y}W - 2H_2\frac{\partial W}{\partial y}}{W^3} w_{ij} = \frac{G_2}{W^3} w_{ij}$$
$$\frac{\partial^2 R_{ij}}{\partial x \partial y} = \frac{\frac{\partial H_1}{\partial y}W - 2H_1\frac{\partial W}{\partial y}}{W^3} w_{ij}$$

where

$$\frac{\partial H_1}{\partial x} = N_i''(x)N_j(y)W - N_i(x)N_j(y)\frac{\partial^2 W}{\partial x^2}$$
$$\frac{\partial H_1}{\partial y} = N_i'(x)N_j'(y)W + N_i'(x)N_j(y)\frac{\partial W}{\partial y} - N_i(x)N_j'(y)\frac{\partial W}{\partial x} - N_i(x)N_j(y)\frac{\partial^2 W}{\partial x \partial y}$$
$$\frac{\partial H_2}{\partial y} = N_i(x)N_j''(y)W - N_i(x)N_j(y)\frac{\partial^2 W}{\partial y^2}$$

Third derivative:

$$\frac{\partial^3 R_{ij}}{\partial x^3} = \frac{\frac{\partial G_1}{\partial x} W - 3G_1 \frac{\partial W}{\partial x}}{W^4} w_{ij}$$
$$\frac{\partial^3 R_{ij}}{\partial y^3} = \frac{\frac{G_2}{\partial y} W - 3G_2 \frac{\partial W}{\partial y}}{W^4} w_{ij}$$
$$\frac{\partial^3 R_{ij}}{\partial x^2 \partial y} = \frac{\frac{\partial G_1}{\partial y} W - 3G_1 \frac{\partial W}{\partial y}}{W^4} w_{ij}$$
$$\frac{\partial^3 R_{ij}}{\partial x \partial y^2} = \frac{\frac{\partial G_2}{\partial x} W - 3G_2 \frac{\partial W}{\partial x}}{W^4} w_{ij}$$

where

$$\frac{\partial G_1}{\partial x} = \frac{\partial^2 H_1}{\partial x^2} W + \frac{\partial H_1}{\partial x}\frac{\partial W}{\partial x} - 2\frac{\partial H_1}{\partial x}\frac{\partial W}{\partial x} - 2H_1 \frac{\partial^2 W}{\partial x^2}$$
$$\frac{\partial G_1}{\partial y} = \frac{\partial^2 H_1}{\partial x \partial y} W + \frac{\partial H_1}{\partial x}\frac{\partial W}{\partial y} - 2\frac{\partial H_1}{\partial y}\frac{\partial W}{\partial x} - 2H_1 \frac{\partial^2 W}{\partial x \partial y}$$
$$\frac{\partial G_2}{\partial x} = \frac{\partial^2 H_2}{\partial x \partial y} W + \frac{\partial H_2}{\partial y}\frac{\partial W}{\partial x} - 2\frac{\partial H_2}{\partial x}\frac{\partial W}{\partial y} - 2H_2 \frac{\partial^2 W}{\partial x \partial y}$$
$$\frac{\partial G_2}{\partial y} = \frac{\partial^2 H_2}{\partial y^2} W + \frac{\partial H_2}{\partial y}\frac{\partial W}{\partial y} - 2\frac{\partial H_2}{\partial y}\frac{\partial W}{\partial y} - 2H_2 \frac{\partial^2 W}{\partial y^2}$$

and

$$\frac{\partial^2 H_1}{\partial x^2} = N_i''' N_j W + N_i'' N_j \frac{\partial W}{\partial x} - N_i' N_j \frac{\partial^2 W}{\partial x^2} - N_i N_j \frac{\partial^3 W}{\partial x^3}$$
$$\frac{\partial^2 H_1}{\partial x \partial y} = N_i'' N_j' W + N_i'' N_j \frac{\partial W}{\partial y} - N_i N_j' \frac{\partial^2 W}{\partial x^2} - N_i N_j \frac{\partial^3 W}{\partial x^2 \partial y}$$
$$\frac{\partial^2 H_2}{\partial y^2} = N_i N_j''' W + N_i N_j'' \frac{\partial W}{\partial y} - N_i N_j' \frac{\partial^2 W}{\partial y^2} - N_i N_j \frac{\partial^3 W}{\partial y^3}$$
$$\frac{\partial^2 H_2}{\partial x \partial y} = N_i' N_j'' W + N_i N_j'' \frac{\partial W}{\partial x} - N_i' N_j \frac{\partial^2 W}{\partial y^2} - N_i N_j \frac{\partial^3 W}{\partial x \partial y^2}$$